

## Appendix 2. Course contents: Calculus as ganita

**Topic 1. Historical background. 1. How Europeans struggled to learn basic math from India.** Why Europeans imported Indian arithmetic (“Arabic numerals”) for large numbers. Inferiority of Greek and Roman pebble arithmetic, lack of large numbers, inefficiency of Greek-Roman arithmetic. How and why Europeans misunderstood Indian place-value arithmetic for centuries: Gerbert and apices, Fibonacci and Florentine law against zero. The terms “zero” and “surd”. When Europeans first learnt fractions and decimal fractions. How ignorance of fractions led to a persistently bad calendar. The related European difficulty of determining even latitude at sea. The Gregorian calendar reform of 1582. How could Europeans have “discovered” calculus at a time when they were still struggling to understand even primary-school arithmetic?

Indian origin of the term “sine”. Not just linguistic confusion, also conceptual confusion: trigonometric functions or circular functions as defined in India? Practical consequences: 7<sup>th</sup> c. Brahmagupta on the need to know the diameter of the earth to determine longitude. How Indians determined the radius of the earth using trigonometry and accurate angle measurements. Its remarkable accuracy. European failure to use trigonometry to determine the radius of the earth, even in the 16<sup>th</sup> c., and persistent European problem of determining longitude at sea. Related 18<sup>th</sup> c. British parliamentary Act setting up a Board of Longitude. In elementary terms, what is  $\sin(120^\circ)$ ? Measurement of angles such as latitude and longitude through arc: radians vs degrees. (2 hours)

**Topic 2. Historical background. 2. Calculus transmission from India to Europe.** How the infinite series claimed to have been “discovered” by Newton (sine series) and Leibniz (infinite series for  $\pi$ ) in 17<sup>th</sup> c. were identical to infinite series known in India since 14<sup>th</sup> c. Madhava of Sangamagrama? Could this identity have been a striking case of independent rediscovery just when dependent discovery was possible due to increased presence of Europeans in India since 15<sup>th</sup> c. Vasco da Gama? The doctrine of Christian discovery and other systematically false historical claims since the Crusades. The evidence for theft of calculus from India on the standard of criminal law (proof beyond reasonable doubt). **Opportunity:** Jesuits in Kochi and the Toledo model. **Motivation:** the European navigational problems, e.g. bad calendar and latitude problem. **Documentary evidence** of prominent Jesuits searching for Indian calendrical sources in Kochi prior to the Gregorian calendar reform of 1562.

Calculus and precise trigonometric values. Trigonometric values, size of the earth and the European longitude problem. **Circumstantial evidence**, such as Fermat’s challenge problem, a problem which no European could solve, but is a solved exercise in Bhaskara’s Bijaganita. Nilakantha planetary model is identical with the Tyconic model etc.

Present-day relevance of correcting the history of calculus origins: stolen knowledge is often misunderstood. How to catch a knowledge thief: **the epistemic test**. Failure to understand as conclusive proof of knowledge theft when there are grounds for suspicion. Did Newton and Leibniz and other European “discoverers” of calculus understand the calculus stolen from India? Do we now understand it in the post-colonial era which has globalized the European misunderstanding of calculus? Should we blindly accept the claims of Western supremacy in mathematics and calculus teaching without open public debate? (4 hours)

**Topic 3. How calculus originated in India.** Geometrical vs numerical understanding of trigonometric functions. Present-day teaching of 6 trigonometric values vs Aryabhata’s and Madhava’s 24 sine values and Vateshwara’s 96 values: progress or regress? Geometry works

only in cases where there is symmetry, numerics works everywhere. Why sine differences (*khandajya*) not sine values? Sine differences and the elementary rule of three (cross-multiplication) as linear interpolation. Calculating  $\sin(1^\circ)$  in various ways. Aryabhata's recursive method vs "Euler" method. Finite differences vs derivatives: chord vs tangent. What is the exact definition of tangent? Approximations and myth of perfection. Introduction to zeroism. (3 hours)

**Topic 4. Differential equations basics.** Relation between values and differences: the "fundamental theorem of calculus". How it changes with varying definitions of derivative and integrals, e.g. Schwartz derivative and Lebesgue integral. Observation that sine differences are proportional to cosine. Observation about second sine differences. Differential equations vs difference equations. Differential equation for the sine function. Solving the differential equations to calculate sine values. Extending this technique: solving differential equations for the exponential function. Using CALCODE. Constructing your own solver. Aryabhata-Euler method. (3 hours)

**Topic 5. Applications of ordinary differential equations.** All problems of Newtonian physics involve the solution of ordinary differential equations. Example problems: (1) Simple harmonic motion. (2) Derivation of the equations of motion of the simple pendulum and conversion to Jacobi's form. The problem of calculating its time period, elliptic integrals and dependence of time period of the simple pendulum on amplitude. (3) The equations of ballistics with air resistance. At what angle should one throw a javelin? A cricket ball? A tennis ball? (4) Solving the 2-body problem of Newtonian gravitation, rosette vs concentric circles/ellipses Example of calculating the trajectory of real spacecraft using NASA Horizons interface. (5) Examples of chaotic motion: the Brusselator as a chemical clock, and the Lorentz model. Various other problems. (These equations will be solved in the practical part of the course.) (6 hours)

**Topic 6. The axiomatic method, number systems and limits.** The origin of formal real numbers: the "error" in the proof of the first proposition of the *Elements*. Dedekind cuts. The problems of Cantorian and naive set theory. Russell's paradox. Limitations and drawbacks of the axiomatic method. Contrast with ganita which accepts empirical proof. Absence of axiomatic proofs in the Euclid book. Where then did axiomatic proof originate? Theological origins of the faith-based axiomatic method and the political reasons for adopting it during the Crusades. Why the axiomatic method does not lead to valid knowledge. Why axiomatic proofs are MORE fallible than empirical proofs. (4 hours)

**Topic 7. Infinitesimals.** Non-Archimedean arithmetic and how infinitesimals are defined on the axiomatic method. Axiomatic algebra, integral domains, fields, field of quotients, ordered fields. Integers and polynomials, rational numbers and rational functions. The integral domain of polynomials with rational coefficients. Extension to field of quotients, rational functions. The sole difference: ordering in polynomials vs ordering in integers. An example of a non-Archimedean ordered field. Limits and Cauchy sequences. Formal reals as the largest Archimedean ordered field. Nonstandard reals vs rational functions. Infinitesimals and non-Archimedean arithmetic in ganita: Brahmagupta's *avyakta ganita*. Solution of quadratic equations. Examples of infinitesimals. Formal definition of limits. Non-existence of limits in a non-Archimedean ordered field. Completeness and limits. Why completeness is NOT needed for calculus contrary to the belief of mathematical analysis. The philosophy of inexactitude. Obtaining limits by discarding infinitesimals, example of infinite geometric series. Relation to limits by order-counting. Irrelevance of limits for practical applications of calculus since Aryabhata and Newton. c

**Topic 8. Practical computation.** All practical value of calculus comes from numerical solution of differential equations, today done on computers. Computers and floating-point

numbers. Introduction to floating-point arithmetic. Floats do not constitute a field: failure of associative law for addition. Doubles. Extended precision arithmetic: limitation of finite memory, infinite precision arithmetic is not possible in finite time. The problem with ints on a computer. (2 hours)

**Topic 9. Higher order polynomial interpolation.** Brahmagupta-Vateshwar-Stirling formula and quadratic interpolation. Its advantages. Runge-Kutta methods and higher order polynomial interpolation. The 12<sup>th</sup> order interpolation extrapolation for precise trigonometric values. Madhava-Taylor series. The Madhava-Leibniz series for  $\pi$ . Its “rate of convergence”. Enhancing precision, accelerating “rate of convergence”. (2 hours)

**Topic 10. (Audit only) Brief introduction to advanced applications.** Shocks and singularities. Brief introduction to shocks as discontinuities. Choosing the differential equations of physics at a shock, Riemann’s error and the Rankine-Hugoniot jump conditions. Shocks and singularities in general relativity. Difficulty of integral formulation of conservation laws in general relativity. Brief intro to the renormalization problem of quantum field theory in configuration space. Products with the Dirac delta. (2 hours)

**Tutorial + Practical:** Apart from classroom lectures, the course will require the use of laptops desktops by the students, to actually solve and analyse a variety of differential equations, since the course teaches calculus as the solution of differential equations of physics, not as symbolic manipulations of integrals and derivatives of elementary functions or as theorems of mathematical analysis with axiomatic real numbers. Introduction to CALCODE, symbolic entry of differential equations, initial data, solution and analysis. Extrema and sections, phase plots. (1) Simple harmonic motion and calculation of  $\pi$ . (2) The simple pendulum. Elliptic integrals and calculating the precise time period of the simple pendulum. (3) Ballistics, path of projectile with air resistance. Distance travelled, and angle of launch. How they vary with resistance. (4) Two body problem. Importance of barycentric motion to explain tidal forces, Tracking satellites, obtaining data from NASA horizons. (5) Chaotic motion, Lorentz model. Brusselator. (10 hours)

**Symbolic manipulation.** Introducing MAXIMA. Using MAXIMA for high precision arithmetic calculations. Using MAXIMA for symbolic manipulation and calculating symbolic derivatives and integrals. Elliptic integrals. (2 hours)

## Textbooks

1. Raju, C. K. *Cultural Foundations of Mathematics: The Nature of Mathematical Proof and the Transmission of the Calculus from India to Europe in the 16th c. CE*. Pearson Longman, 2007.
2. Raju, C. K. *Euclid and Jesus: How and Why the Church Changed Mathematics and Christianity across Two Religious Wars*. Penang: Multiversity and Citizens International, 2012.

## References

- Āryabhaṭa, *Āryabhaṭīya of Āryabhaṭa*, ed. K. S. Shukla and K. V. Sarma (Delhi: Indian National Science Academy, 1976).
- Brahmagupta, *Brahma-Sphuta Siddhanta*, ed. Ram Swarup Sharma (New Delhi: Indian Institute of Astronomical and Sanskrit Research, 1966). Chapter 18.
- H.J. Keisler, *Elementary Calculus: An Approach Using Infinitesimals* (Boston: Prindle, Weber & Schmidt, 1971).
- Walter Rudin, *Principles of Mathematical Analysis* (New York: McGraw-Hill, 1976).

## Additional references

1. Raju, C. K. 'Calculus'. In *Encyclopedia of Non-Western Science, Technology and Medicine*, edited by Helaine Selin, 1010–15. Dordrecht: Springer, 2016. <http://ckraju.net/papers/Springer/ckr-Springer-encyclopedia-calculus-1-final.pdf>.
2. Raju, C. K. 'Calculus Transmission'. In *Encyclopedia of Non-Western Science, Technology and Medicine*, edited by Helaine Selin, 1016–22. Dordrecht: Springer, 2016. <http://ckraju.net/papers/Springer/ckr-Springer-encyclopedia-calculus-2-final.pdf>.
3. Raju, C. K. 'Teaching Mathematics with a Different Philosophy. 1: Formal Mathematics as Biased Metaphysics'. *Science and Culture* 77, no. 7–8 (2011): 274–79. arXiv:1312.2099.
4. Video of lecture at MIT, Cambridge, Mass.: "Calculus: the real story" <https://www.youtube.com/watch?v=laodCGDjqzs>. Presentation: <https://ckraju.net/papers/presentations/MIT.pdf>. Abstract: <http://ckraju.net/papers/Calculus-story-abstract.html>.
5. Videos: (a) C. K. Raju. 'Ganita vs Formal Math: An Obituary of Formal Math'. Indian Institute of Advanced Study, Shimla, 25 March 2021. <https://www.youtube.com/watch?v=0UrmypVN6c0>. (b) *Vimarsh on Practical Gaṇita vs Religious Mathematics by Prof CK Raju, PhD (ISI), TGA Laureate*, Vivekanand Foundation, 2023. <https://www.youtube.com/watch?v=zTqL9EkENil>. (c) *Ganita vs Math | Inferiority of Western Math*, 2023. <https://www.youtube.com/watch?v=c0-rPGDTd4c>
6. Raju, C. K. 'Products and Compositions with the Dirac Delta Function'. *J. Phys. A: Math. Gen.* 15 (1982): 381–96. Raju, C. K. 'On the Square of  $x^{-n}$ '. *Journal of Physics A: Mathematical and General* 16 (1983): 3739–53. <https://doi.org/10.1088/0305-4470/16/16/013>. Raju, C. K. 'Distributional Matter Tensors in Relativity'. In *Proceedings of the 5th Marcel Grossman Meeting*, edited by D. Blair and M. J. Buckingham, 421–23. World Scientific, 1989. [arXiv:0804.1998](https://arxiv.org/abs/0804.1998). Raju, Suvrat, and C. K. Raju. 'Radiation Damping and Functional Differential Equations'. *Mod. Phys. Lett. A* 26, no. 35 (2011): 2627–38.
7. General decolonization reading list: <https://tinyurl.com/decol-list-new>.